Composite Peak Modulation of a Compatible SSB Stereo Encoder

Robert Orban
Chief Engineer
Orban

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Recently, the broadcast industry has become interested in the possibility of using single sideband modulation in the FM stereo subchannel to replace standard double-sideband suppressed carrier modulation. Using certain assumptions, it has been suggested that conventional radios are compatible with SSB modulation. These assumptions include synchronous detection of the FM subcarrier and insensitivity of the receiver to the quadrature component of the SSB modulation. The author has commented about these issues in another article that has also been made available to the NRSC.

Another significant consideration is the peak modulation produced by the SSB waveform compared to the peak modulation produced by conventional DSB modulation. The DSB system has an important characteristic commonly called "interleaving." In the absence of the stereo pilot tone, the peak modulation of the composite waveform is the larger of the left or right audio channels. This allows the peak modulation of the composite waveform to be well controlled if the left and right channels are peak-limited separately and independently. Interleaving is a fortuitous consequence of the fact that the DSB system can also be considered to be a system that alternately samples the left and right input signals at a 38 kHz rate. This can be demonstrated by using a few trigonometric identities.

The stereo pilot tone slightly changes this ideal performance. Because it is correlated to the stereo subcarrier, it can be shown that in the presence of the pilot tone, L+R modulation (i.e., where equal in-phase signals peak-limited to 100% modulation are applied to the left and right inputs of the stereo modulator) produces approximately 2.7% higher peak modulation than does left-only or right-only modulation, assuming 9% pilot injection.

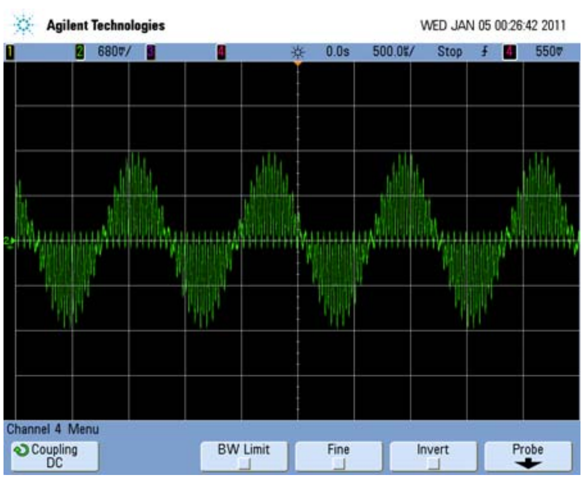

Figure 1

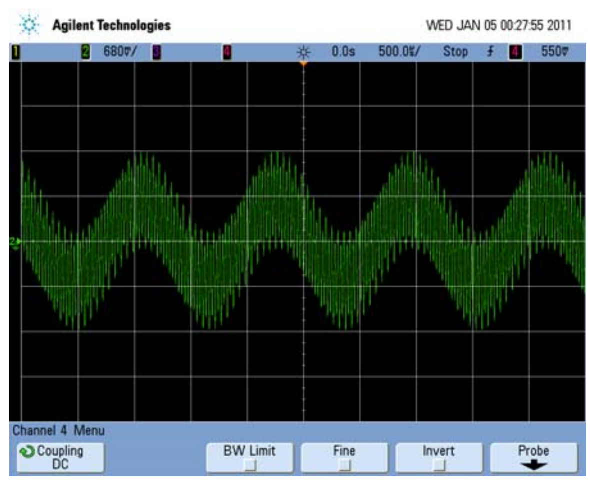

Figure 2

The peak modulation behavior of the SSB system is more complicated. Figures 1 and 2 show the composite waveform of left-channel-only modulation for the DSB and SSB cases respectively.

In the SSB case, the stereo subchannel has only one spectral component, so the envelope of the stereo subchannel is essentially flat and rides on top of the L+R component, which is also a single sinewave. The DSB case shows the interleaving property: the single-channel modulation is essentially “sampled” so the peak modulation is the same as the left channel. The values of the SSB peak modulation in this case is the same as that of the DSB modulation. However, we will show below that this is not always true.

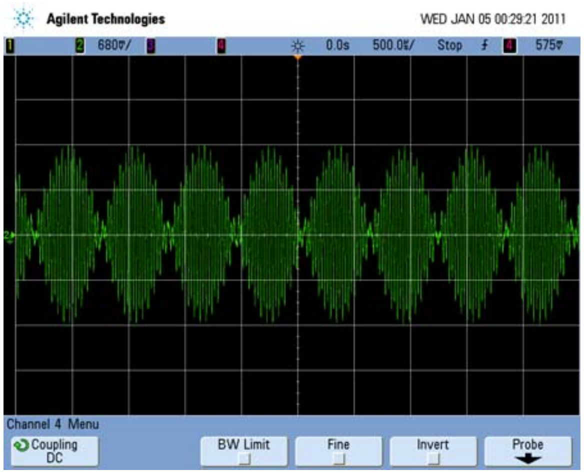

Figure 3

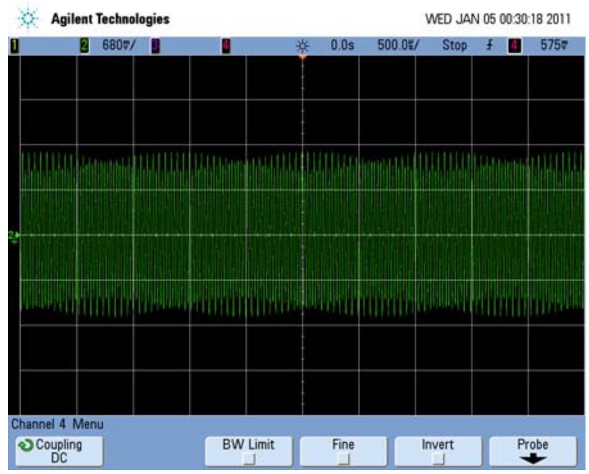

Figure 4

Figures 3 and 4 show RIGHT = –LEFT modulation (L–R) for the DSB and SSB cases respectively. The DSB shows the interleaving property, while the SSB case shows a single sinewave beating with the pilot tone. Once again, peak modulation is the same.

The next set of pictures shows the effect of applying sinewaves of the same frequency and amplitude to the left and right inputs, but where the right channel input is phase-shifted compared to the left channel input. To avoid spurious results caused by a peak limiter in the system used to generate these waveforms, the level of the left and right modulation was reduced to

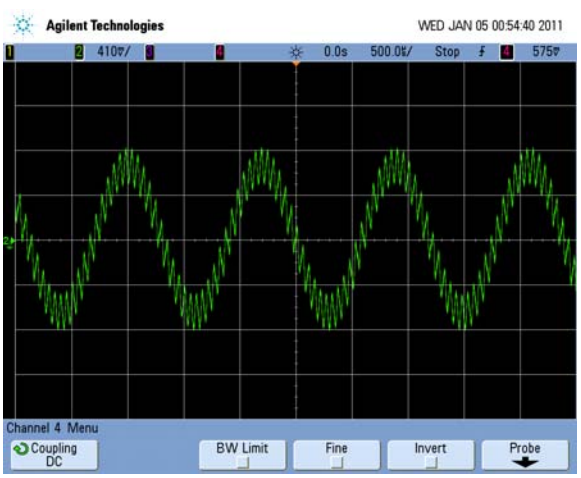

Figure 5

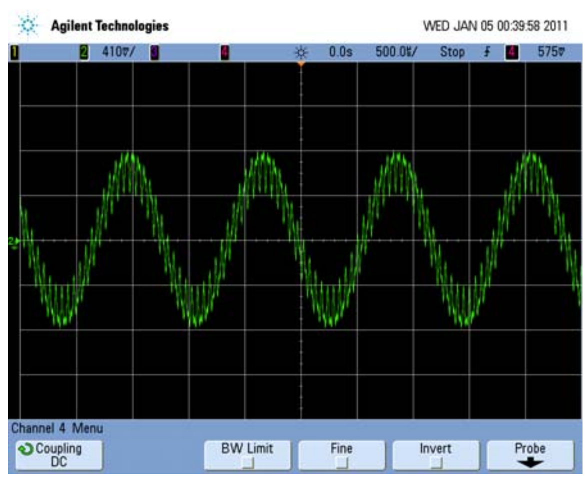

Figure 6

approximately 50% and the sensitivity of the oscilloscope was increased to compensate for this. In essence, this increases the pilot injection to about 18% but allows us to see what happens as the phase shift is varied.

Figures 5 and 6 show in-phase modulation. As expected, the L–R energy is zero and the two cases are the same.

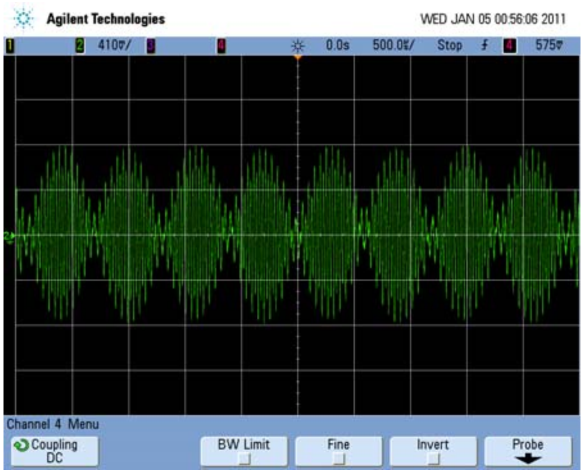

Figure 7

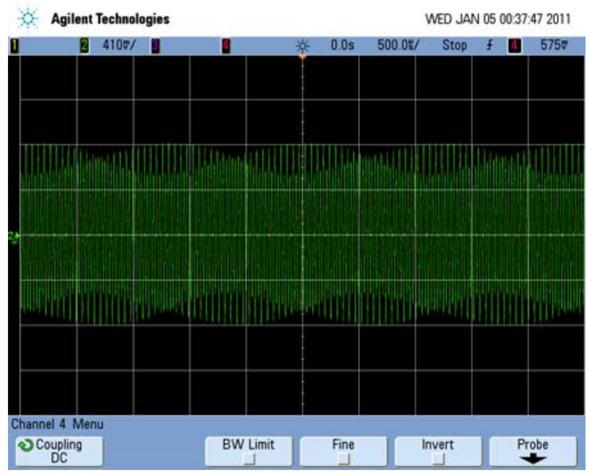

Figure 8

Figures 7 and 8 show modulation with 180 degrees of phase shift between the channels. Other than the increased relative pilot injection, this is the same as Figures 3 and 4.

Figures 9 and 10 show peak modulation with approximately 90 degrees of phase shift between the left and right inputs. While the modulation in the DSB remains at 100% (as expected), something very significant happens in the SSB case—peak modulation increases to approximately 140%. This is because the peak modulation is essentially the sum of the L+R and the envelope of the L–R signals. For phase shifts other than 0 degrees and 180 degrees, this sum

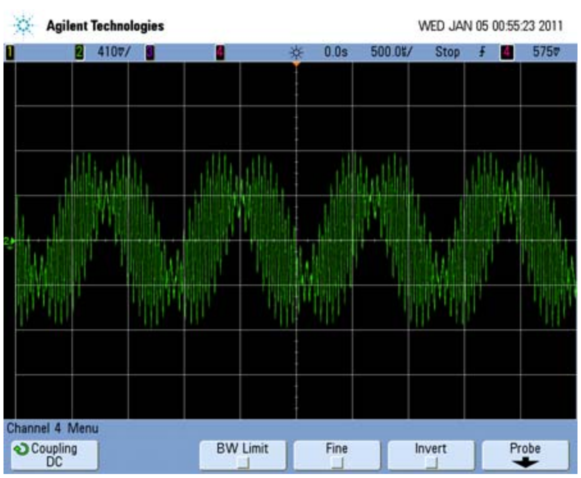

Figure 9

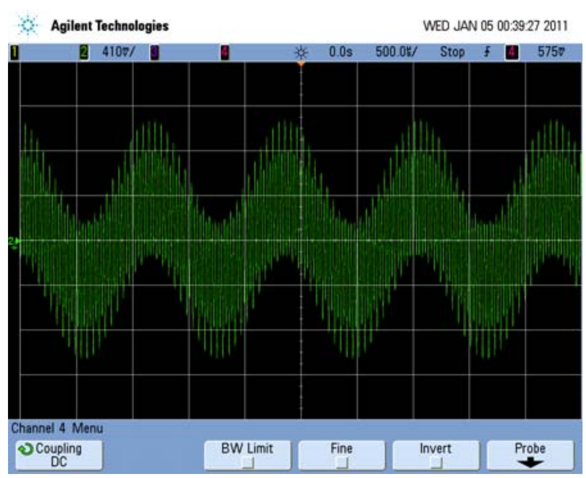

Figure 10

exceeds 100%.

We can examine this behavior mathematically by modeling the L+R and L–R signals as the sum of two phasors representing the left and right channels. Figure 11 shows this graphically, where the left channel and right channel are of equal magnitude (as would be the case if they were correctly peak-limited for DSB stereo) but the right channel is phase-shifted by φ radians with respect to the left channel.

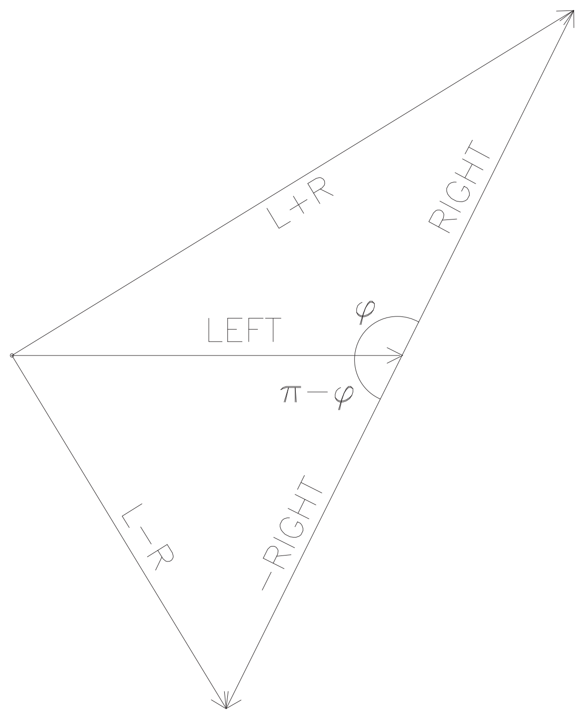

Figure 11

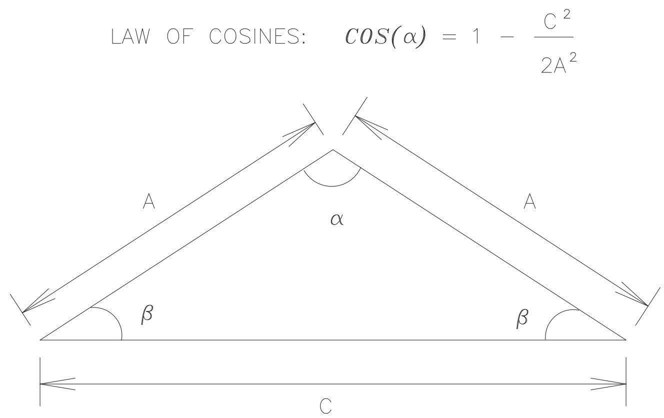

Figure 12

It is obvious from the diagram that –RIGHT is phase-shifted by π – φ radians. The L+R and L–R vectors are the sum of left and right and left and –right vectors respectively.

We wish to calculate the value of φ that maximizes the peak modulation of the SSB composite waveform for the specific left and right input signals we are analyzing. As is clear from the scope photos above, the worst-case peak modulation of the SSB composite is the sum of L+R and the envelope of the frequency-shifted and spectrally inverted L–R signal that the SSB modulator places in the stereo subchannel. This signal is a sinewave with the same magnitude as the L–R signal. The highest peak modulation occurs when the peaks of the signal in the stereo subchannel are time-coincident with peak of the L+R signal. It is therefore safe to use the peak value of the L–R signal as the worst-case value of the envelope of the stereo subchannel.

Let us formulate this problem mathematically. The vector diagram in Figure 11 contains two isosceles triangles, one of which is formed by LEFT/RIGHT/L+R and the other of which is formed by LEFT/–RIGHT/L–R. Figure 12 shows the well-known Law of Cosines for a isosceles triangle, which will allow us to compute the magnitude of the L+R and L–R signals as a function of φ.

Without loss of generality (because we intend to express overshoots as a percentage of the left and right peak modulation), we will normalize the calculation by setting the peak magnitudes of the left and right signals to 1.

Referring to Figure 12, the Law of Cosines can be rearranged as:

#1: $c^2 = 2 \cdot a^2 \cdot (1 - \cos(\alpha))$

It is well known that

#2: $\cos(\pi - \phi) = -\cos(\phi)$.

Using EQ (1) and the identity in EQ (2), we can now write an equation for the sum of the magnitudes of L+R and L–R:

#3: $\text{magnitude} = \sqrt{(2 \cdot (1 - \cos(\phi)))} + \sqrt{(2 \cdot (1 + \cos(\phi)))}$

To determine the peak value of the magnitude, we differentiate it with respect to φ and set the result equal to zero. At least one zero of this equation will be a value of φ that maximizes `magnitude`.

#4: $\frac{d}{d\phi} \left(\sqrt{(2 \cdot (1 - \cos(\phi)))} + \sqrt{(2 \cdot (1 + \cos(\phi)))}\right) = 0$

This can be simplified as:

#5: $$\frac{\sqrt{2} \cdot \sin(\phi)}{2 \cdot \sqrt{(1 - \cos(\phi))}} - \frac{\sqrt{2} \cdot \sin(\phi)}{2 \cdot \sqrt{(\cos(\phi) + 1)}} = 0$$

This equation has several principal solutions:

#6: $$\phi = \frac{3 \cdot \pi}{2} \vee \phi = -\frac{\pi}{2} \vee \phi = \frac{\pi}{2}$$

These solutions are all odd integer multiples of $\pi/2$, or 90 degrees.

We substitute $\pi/2$ for ϕ in the expression for the peak magnitude

#7: $\text{magnitude} = \sqrt{\left(2 \cdot \left(1 - \cos\left(\frac{\pi}{2}\right)\right)\right)} + \sqrt{\left(2 \cdot \left(1 + \cos\left(\frac{\pi}{2}\right)\right)\right)}$

This simplifies as:

#8: $$\text{magnitude} = 2 \cdot \sqrt{2}$$

indicating that the magnitudes of L-R and L+R are both equal to $\sqrt{2}$, as expected from the Pythagorean Theorem, which is applicable when $\phi = \pi/2$.

For the case of $\phi = 0$, we have

#9: $\text{magnitude} = \sqrt{(2\cdot(1 - \cos(0)))} + \sqrt{(2\cdot(1 + \cos(0)))} = 2$

For the case of $\phi = \pi$, $\text{magnitude} = 2$ as well.

We therefore conclude that the worst-case peak overshoot occurs at $\phi = \pi/2$ and is

#10: $2\sqrt{2} / 2 = \sqrt{2}$.

This does not include the effect of the pilot tone.

Assuming 9% injection to achieve 100% modulation, the peak modulation without pilot tone is 91% for $\phi = 0$ and $\sqrt{2}$ 91% for $\phi = \pi/2$.

#11: $0.91\cdot\sqrt{2} + 0.09 = 1.3769\ldots$

We therefore conclude that the worst-case peak overshoot for two sinewaves of equal magnitude but unequal phase is 138% modulation including 9% pilot tone. (This calculation did not take into account any effect due to the pilot tone's being phase-locked to the subcarrier.)

The author cannot deduce from the calculation above if there are other signals that can cause more than 138% modulation. However, the case of two sinewaves is cause for serious concern because it shows controlling the composite modulation can require the audio processor to reduce the peak modulation of the composite by as much as 2.8 dB compared to the DSB case. This amount of peak limiting will almost certainly cause audible artifacts, particularly if it is applied to audio signals that are already likely to have received considerable amounts of peak limiting in the discrete left and right domain.

This restriction may be eased if the regulatory authority changes modulation regulations to allow greater than ±75 kHz peak deviation of the RF carrier because measurements have shown that the RF bandwidth produced by the SSB modulation is lower than that produced by conventional DSB modulation. Determining whether this is practical will require adjacent-channel RF protection ratio measurements with program material that causes the SSB modulation to produce large overshoots such as those calculated above. In addition, some attention should be given to the question of whether some receivers whose RF and IF subsystems were designed assuming ±75 kHz peak deviation will add significant distortion when presented with wider deviation.